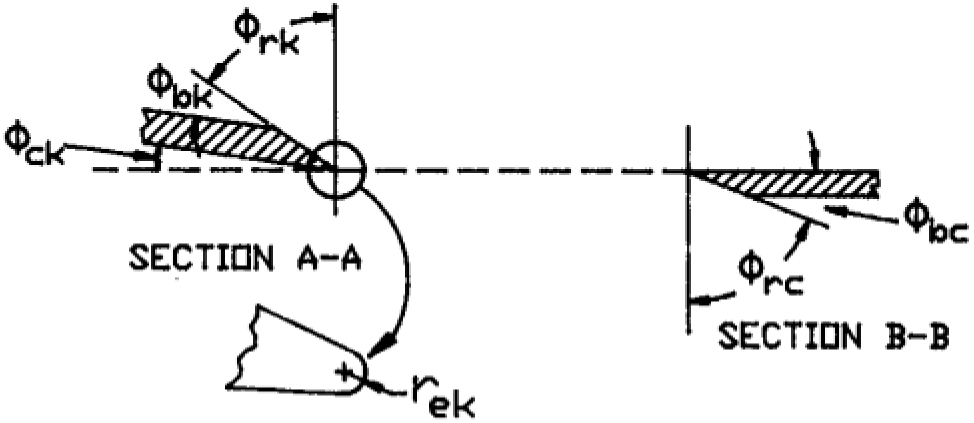
It is common knowledge that a sharp knife aids cutting. It is important, however, to distinguish between *sharpness* and *fineness*. A fine knife has a small bevel angle, ϕ_{bk}, while a blunt knife has a large bevel angle. Sharpness is defined by the edge radius, r_{ek}, of the knife, i.e., a sharp knife has a small radius while a dull knife has a larger radius. Initial penetration of the knife into the plant material is aided if the knife rake angle, ϕ_{rk}, is large. The knife clearance angle, ϕ_{ck}, is the angle formed between the bottom edge of the knife and the x-y plane. In general, the following relationship holds between the rake, bevel, and clearance angles:



$$\varphi_{rk} + \varphi_{bk} + \varphi_{ck} = 90°$$

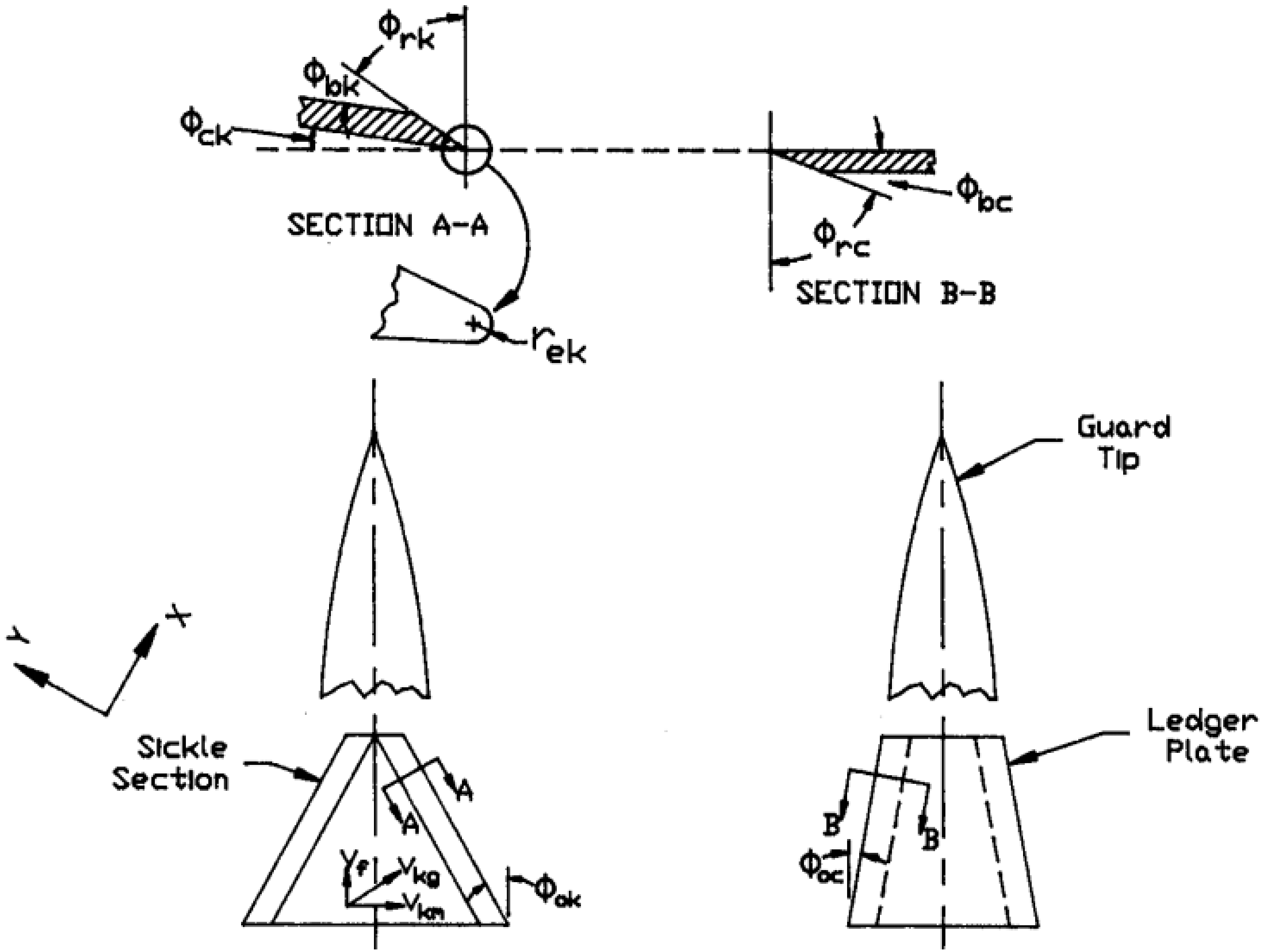


Illustration of geometry of a knife and countershear.

the chip angle on the knife, φ_{chk}, is defined as follows:

$$\varphi_{chk} = \varphi_{bk} + \varphi_{ck}$$

The bevel, rake, clearance, and oblique angles all have their counterparts on the countershear, as shown in Figure 11.8. For each of these angles, the subscript k indicates that it relates to the knife, while a subscript c indicates the corresponding angle

on the countershear. The clip angle, φcl, is the angle formed between the knife and countershear, i.e.:

$$\varphi_{cl} = \varphi_{ok} + \varphi_{oc}$$

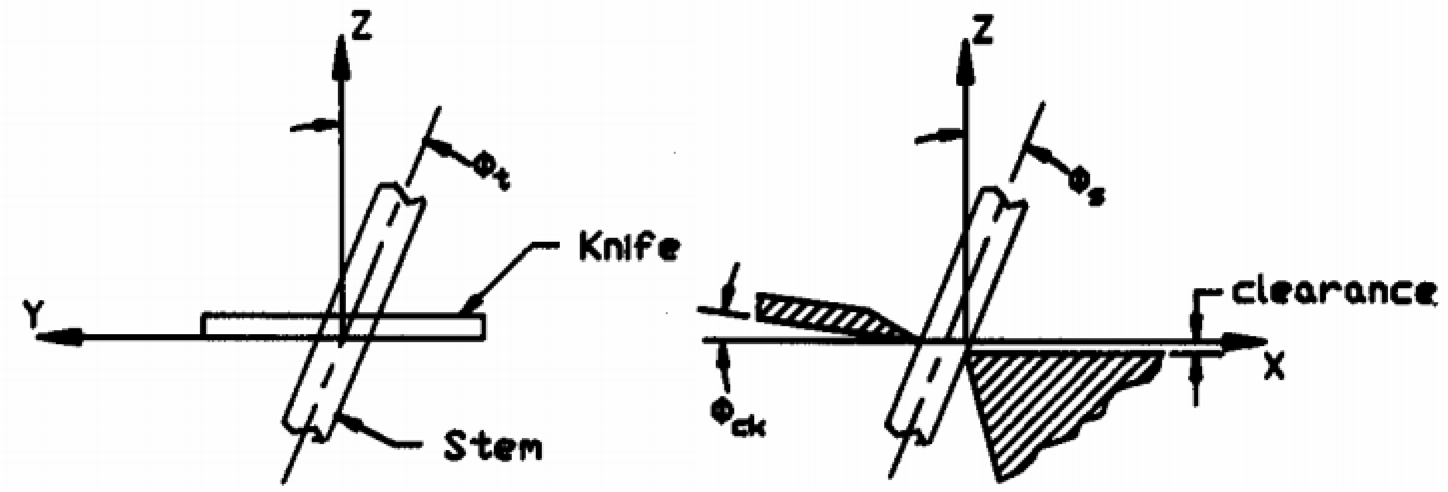


Illustration of stem misalignment.

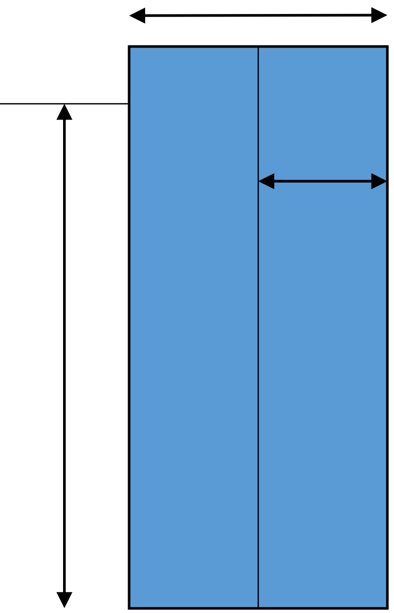
When ϕ_{ok} is not zero and the plant material is not yet in contact with the countershear, the possibility exists that the plant material may slide along the edge of the knife before or while being cut. The sliding is expected if the oblique angle is greater than the following maximum angle:

$$\phi_{ok,max} = \arctan f_{ek} \tag{11.4}$$

where f_{ek} = knife edge friction coefficient. The knife edge friction coefficient is the lateral force (parallel to the knife edge) imposed by the plant on the knife edge divided by the normal force imposed by the plant. When the plant is in contact with both the knife and countershear, sliding is expected if the clip angle is greater than the following maximum:

$$\phi_{cl,max} = \arctan \frac{f_{ek} + f_{ec}}{1 - f_{ek} f_{ec}} \tag{11.5}$$

where $\phi_{cl,max}$ = maximum value of ϕ_{cl} that will prevent sliding
f_{ek} = friction coefficient for knife edge
f_{ec} = friction coefficient for countershear edge



The bending strength of a plant stem may be important during cutting. For example, some devices cut a plant in the absence of a countershear; the plant stem below the cutting plane is loaded as a cantilever beam. In other situations, the stem may be loaded as a simply supported beam. In either case, the direction of loading is radial (perpendicular to the longitudinal axis of the plant stem). The radial load that would cause failure in bending can be calculated using the following equation:

$$F_{bu} = \frac{I}{c}\frac{S_u}{L} \qquad (11.6)$$

where F_{bu} = ultimate load at bending failure, N

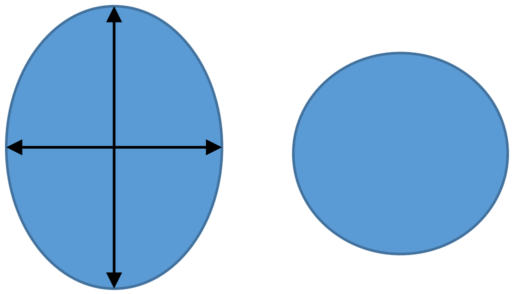
I = moment of inertia of the cross section, mm^4

c = radius from neutral axis of stem to most distant load-carrying fiber, mm

or, alternately, I/c = section modulus, mm^3

S_u = ultimate stress of plant fibers, N/mm^2

L = distance from concentrated load to point of support, mm



The deflection of the stem is given by:

$$\delta_r = \frac{F_r L^3}{C_b E I} \tag{11.7}$$

where δ_r = radial deflection, mm

F_r = radial concentrated load, N

E = modulus of elasticity of stem fibers, N/mm^2

C_b = constant (3 for cantilevered stems, 48 for simply supported stems).

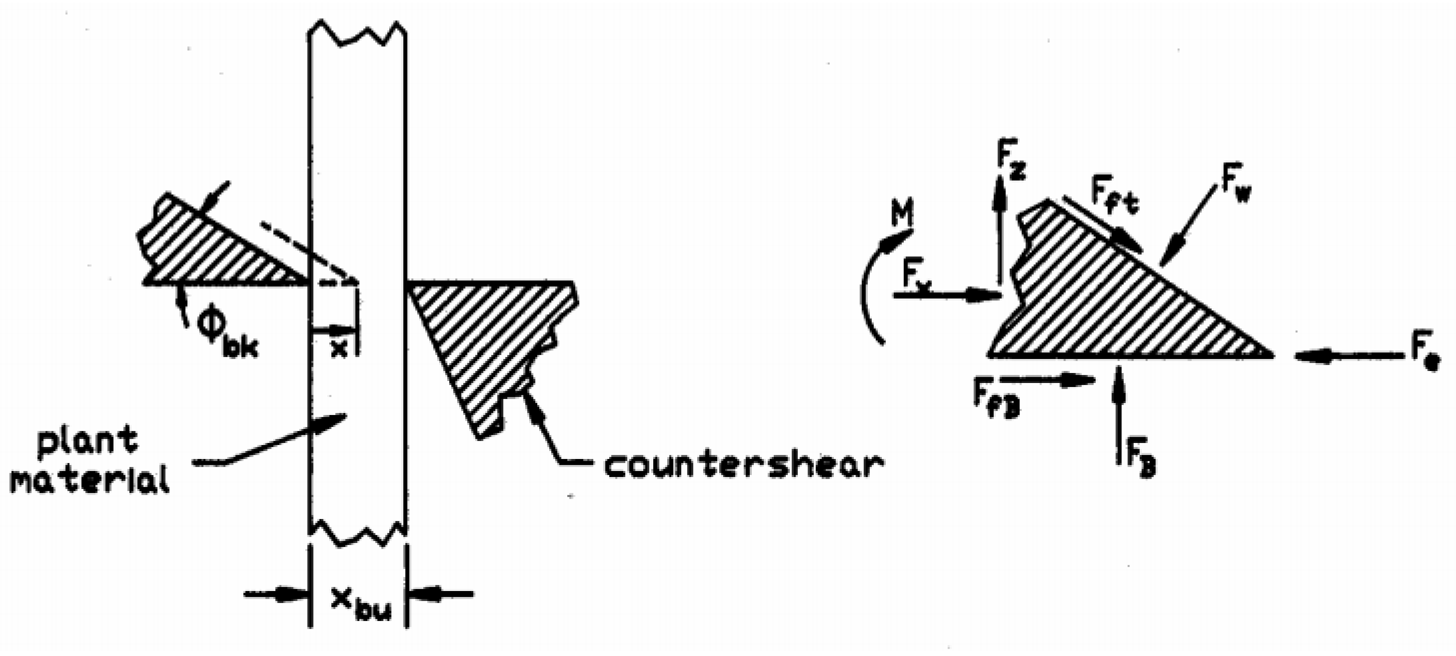
The moment of inertia of a homogeneous solid, circular section is:

$$I = \frac{\pi d^4}{64} \tag{11.8}$$

where d = diameter of the section, mm. For a hollow, thin-walled section, the moment of inertia is:

$$I = \frac{3\pi d^3 t}{32} \tag{11.9}$$

where t = wall thickness, mm. From comparing Equations 11.8 and 11.9, we note that the moment of inertia of a natural stem should be proportional to the section diameter raised to an exponent between 3 and 4.



(a) (b)

Illustration of knife forces during cutting knife motion is the sum of the knife edge force plus x-components of forces imposed on the top and bottom surfaces of the knife as it compresses and penetrates the plant material. By assuming that only the material directly ahead of the knife is compressed and by use of the bulk modulus for the plant material, the following equation for knife force was derived:

$$\frac{F_x}{w} = \frac{F_{ek}}{w} + \frac{B_f x^{\lambda}}{2X_{bu}}(\tan\phi_{bk} + 2f) \tag{11.10}$$

where F_x = knife driving force in x direction, N

F_{ek} = force imposed by plant on the knife edge, N

w = width of knife, mm

B_f = bulk modulus of forage, N/mm^2

x = knife displacement after initial contact, mm

λ = exponent

X_{bu} = uncompressed depth of material between knife and countershear, mm

ϕ_{bk} = bevel angle of knife edge

f = coefficient of friction of forage on knife

Theoretically, $\lambda = 2$ in Equation 11.10. However, a smaller exponent gives a better fit to some experimental cutting data. In an experiment in which a thin (X_{bu} = 8.9 mm) bed of timothy at 20% moisture was cut at a very low (0.42 mm/s) knife speed, Equation 11.10 fit the data when $\lambda = 1.46$ and $B_f = 10\ N/mm^2$. The knife edge force is calculated as the product of the projected frontal area of the knife edge times the pressure imposed on that edge by the forage. The approximate frontal area can be calculated from the following equation:

$$A_{ek} = r_{ek}\left[1 + \cos(\phi_{bk} + \phi_{ck})\right] \tag{11.11}$$

where A_{ek} = frontal area of knife edge per mm of width, mm^2

r_{ek} = radius of knife edge, mm

Figure 11.15 shows the typical shape of the force-displacement curve when plant material is cut by a knife and countershear. In Section A, only compression occurs as the knife edge force is not yet high enough to cause cutting. After initial stem failure, some compression continues in Section B along with cutting. In section C, the material is fully compressed; cutting continues and then the force drops rapidly as the knife edge crosses the edge of the countershear. With suitable choice of parameters, the force in Sections A and B could be calculated using Equation 11.10. Section C does not involve compression so Equation 11.10 does not fit that section. The diagram in Figure 11.15 is for a straight cut, i.e., with $\phi_{cl} = 0$. For an oblique cut, the peak cutting force would be reduced and the duration of cut would be extended compared to Figure 11.15.

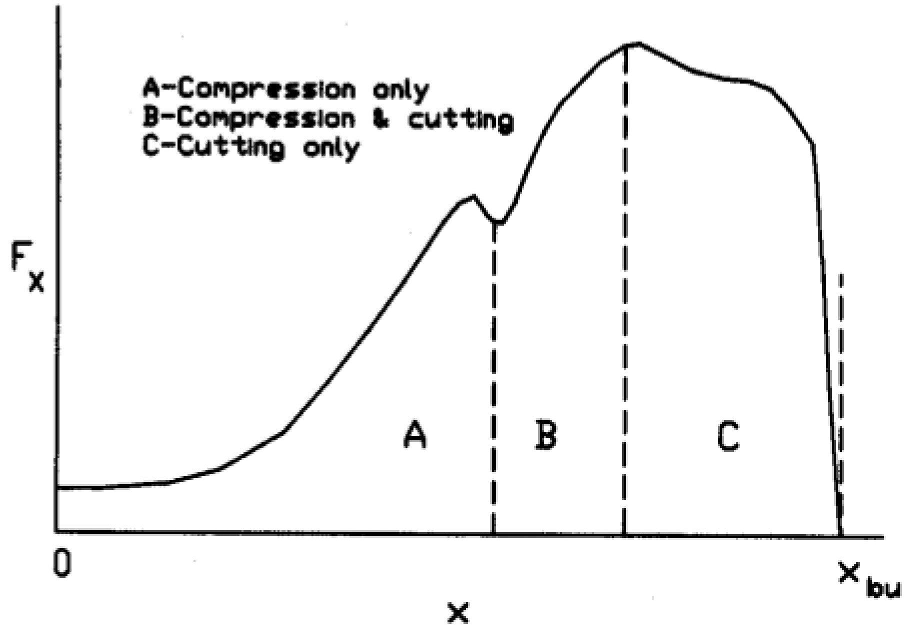


Figure 11.15 – Knife force-displacement curve for a straight cut against a countershear.

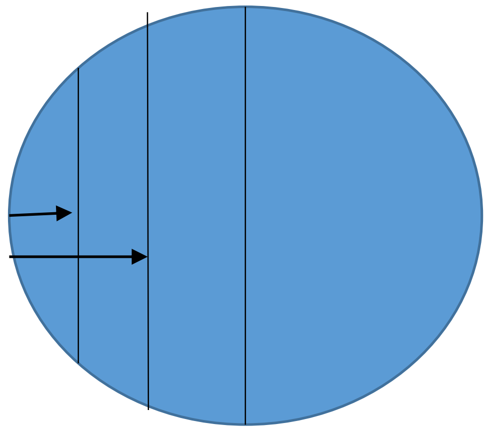


Figure 11.15 is useful in calculating the power requirement for cutting with a knife and countershear. The energy per cut is equal to the area under the cutting force curve; multiplying by the cutting frequency gives the power. The following equation can be used to compute the power requirement for cutting:

$$P_{cut} = \frac{C_f F_{max} X_{bu} f_{cut}}{60{,}000} \tag{11.12}$$

where P_{cut} = power for cutting, kW

F_{xmax} = maximum cutting force, kN

X_{bu} = depth of material at initial contact with knife, mm (see Figure 11.15)

f_{cut} = cutting frequency, cuts/min

C_f = ratio of average to peak cutting force

C_f is always be between 0 and 1 and, for a typical force-displacement curve as illustrated in Figure 11.15, it is approximately equal to 0.64.

The cutting force, F_x, must be supported. If a countershear is present and clearance is small, the support force can be provided entirely by the countershear. When no countershear is present, the support force must be provided entirely by the plant itself through the bending strength of the stump below the cut and the inertia of the plant above the cut. The resulting cut is called, alternatively, an impact cut, an inertia cut, or a free cut. As clearance with a countershear increases, the plant strength and inertia come increasingly into play; thus, impact cutting is similar to countershear cutting with very large clearance. Figure 11.16 illustrates the forces and moments on the plant during impact cutting. The soil and the plant root system provide a force, F_B, and a moment, M_r, which tend to keep the stump upright. Acceleration of the stump is considered to be negligible. Force F_B represents the combined effects of the root system and stalk strength in providing bending resistance at the height of the cut. The center of gravity of the cut portion of the plant is at a height, z_{cg}, above the cut. The impact shown in Figure 11.16 tends to accelerate the cut plant to the right and counterclockwise; consequently, an inertia force and inertia moment appear on the plant at the center of gravity. The following equation results from summing moments about the center of gravity of the cut plant:

$$I_p \alpha_p = (F_x - F_b) z_{cg} \tag{11.13}$$

where α_p = angular acceleration of plant, radians/s^2

F_x = cutting force, N

F_b = bending resistance of stump, N

z_{cg} = height of center of gravity of cut plant, m (see Figure 11.16)

I_p = centroidal moment of inertia of plant, kg m^2

= $m_p \, r_g^2$ where m_p = mass of cut portion of plant, kg

r_g = radius of gyration of cut portion of plant, m

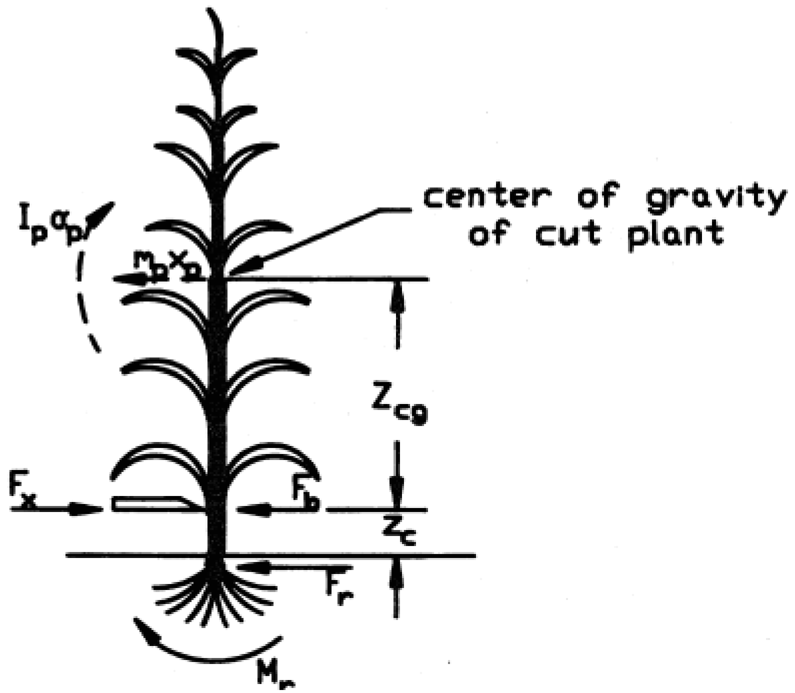


Figure 11.16 – Forces and moments in impact cutting.

An analysis of the kinematics (motions) of the plant gives the following equation for angular acceleration:

$$\alpha_p = \frac{a_c - a_{cg}}{z_{cg}} \tag{11.14}$$

where a_c = acceleration of plant at plane of cut, m/s^2

a_{cg} = acceleration of plant center of gravity, m/s^2

By assuming that the plant acquires knife velocity at the plane of cut, the following equation was derived:

$$a_c = \frac{1000 v_k^2}{d_s} \tag{11.15}$$

where v_k = knife velocity, m/s

d_s = stalk diameter at plane of cut, mm

Equations 11.13 through 11.15 can be combined to give the following equation for minimum knife velocity for impact cutting:

$$v_k = \left[d_s \frac{(F_x - F_b)}{1000 m_p} \left(1 + \frac{z_{cg}^2}{r_g^2} \right) \right]^{0.5} \tag{11.16}$$

When values for r_g and z_{cg} are not readily available, a simpler approximate equation can be obtained by assuming that $r_g = z_{cg}$. The simpler equation illustrates the key variables involved in impact cutting. If the stump bending resistance, F_b, is large enough to support the entire cutting force, F_x, the minimum required knife velocity is zero and cutting is equivalent to cutting with a countershear. Lowering the height of cut to increase F_b and reducing F_x by maintaining a sharp knife will both reduce the minimum required knife velocity. Tests of impact cutting of timothy, for example, have shown that cutting could be accomplished at knife velocities as low as 25 m/s but velocities of 45 m/s were required for reliable cutting of all stems. To assure reliable cutting over a wide range of knife sharpness and stem stiffness, minimum knife velocities of 50 to 75 m/s are generally recommended. Example Problem 11.2 illustrates the calculation of minimum knife velocity for impact cutting.