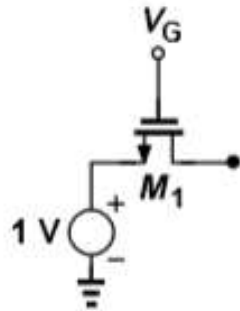# Derivation of I/V Characteristics

- **If $V_{DS} \ll 2(V_{GS}-V_{TH})$, the device is operating in "deep triode region."**

- **In this region, a MOSFET can operate as a resistor whose value is controlled by the overdrive voltage.**

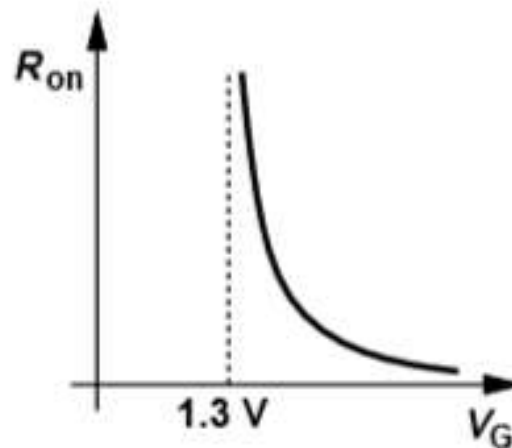- **Unlike bipolar transistors, a MOS device may be on even if it carries no current.**
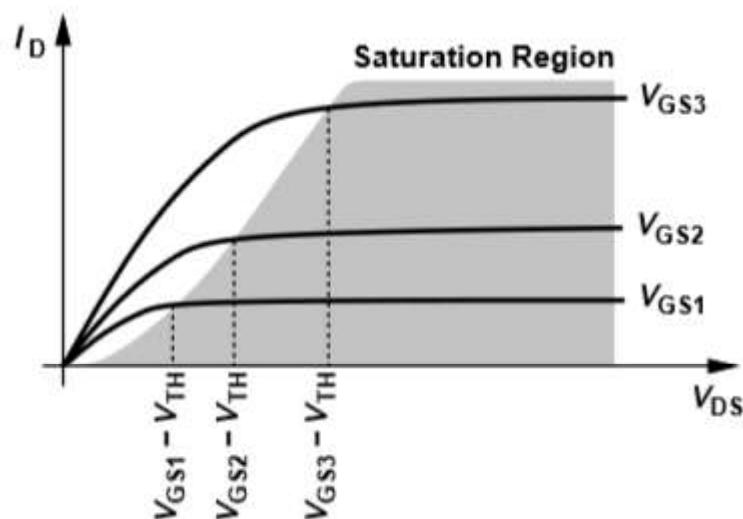
Derivation of I/V Characteristics

- **For example, given the topology on the left and that**
  - $\mu_n C_{ox} = 50 \ \mu A/V^2$,
  - $W/L = 10$,
  - $V_{TH} = 0.3 \text{ V}$,

$$R_{on} = \frac{1}{50 \ \mu A/V^2 \times 10(V_G - 1 \text{ V} - 0.3 \text{ V})}.$$
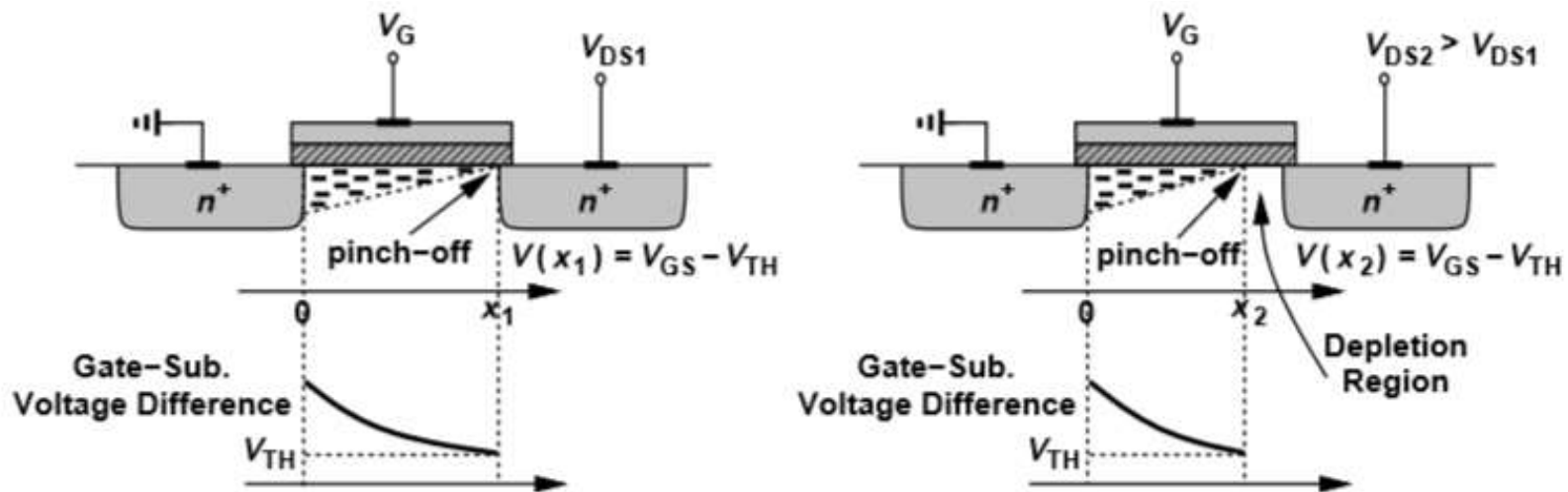
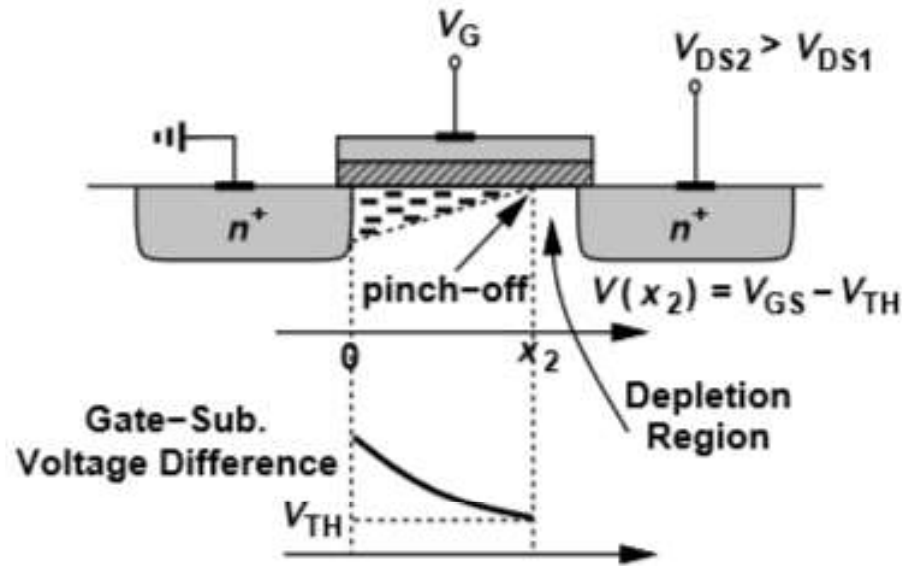# Derivation of I/V Characteristics

- **In reality, if $V_{DS} > V_{GS} - V_{TH}$, I_D becomes relatively constant and we say that the device operates in "saturation region."**

- **$V_{D,sat} = V_{GS} - V_{TH}$ denotes the minimum V_{DS} necessary for operation in saturation.**

Derivation of I/V Characteristics

- **If V_{DS} is slightly larger than $V_{GS}-V_{TH}$, the inversion layer stops at $x \leq L$, and the channel becomes "pinched off."**

- **As V_{DS} increases, the point at which Q_D equals zero gradually moves towards the source.**

- **At some point along the channel, the local potential difference between the gate and the oxide-silicon interface is not sufficient to support an inversion layer.**

Derivation of I/V Characteristics

- **Electron velocity ($v = I/Q_d$) rises tremendously as they approach the pinch-off point (where $Q_d \to 0$) and shoot through the depletion region near the drain junction and arrive at the drain terminal.**

Derivation of I/V Characteristics

$$V_{GS} = \sqrt{\frac{2I_D}{\mu_n C_{ox} \dfrac{W}{L'}}} + V_{TH}$$

- **Since the integral becomes**

$$\int_{x=0}^{x=x_2=L'} I_D \, dx = \int_{V=0}^{V=V_{GS}-V_{TH}} W C_{ox} \mu_n [V_{GS} - V(x) - V_{TH}] dV.$$

$$I_D = \frac{1}{2} \mu_n C_{ox} \left(\frac{W}{L'}\right) (V_{GS} - V_{TH})^2.$$

- **I_D is relatively independent of V_{DS} if L' remains close to L.**

- **The device exhibits a "square-law" behavior.**

Derivation of I/V Characteristics

- **For PMOS devices, the equations become**

  - $I_D = -\mu_p C_{ox} \dfrac{W}{L} \left[(V_{GS} - V_{TH})V_{DS} - \dfrac{1}{2} V_{DS}^2 \right].$

  - $I_D = -\dfrac{1}{2} \mu_p C_{ox} \dfrac{W}{L'} (V_{GS} - V_{TH})^2.$

- **The negative sign shows up due to the assumption that drain current flows from drain to source, whereas holes in a PMOS flow in the reverse direction.**

- **V_{GS}, V_{DS}, V_{TH}, and $V_{GS} - V_{TH}$ are negative for a PMOS transistor that is turned on.**

- **Since the mobility of holes is about ½ the mobility of electrons, PMOS devices suffer from lower "current drive" capability.**

Derivation of I/V Characteristics
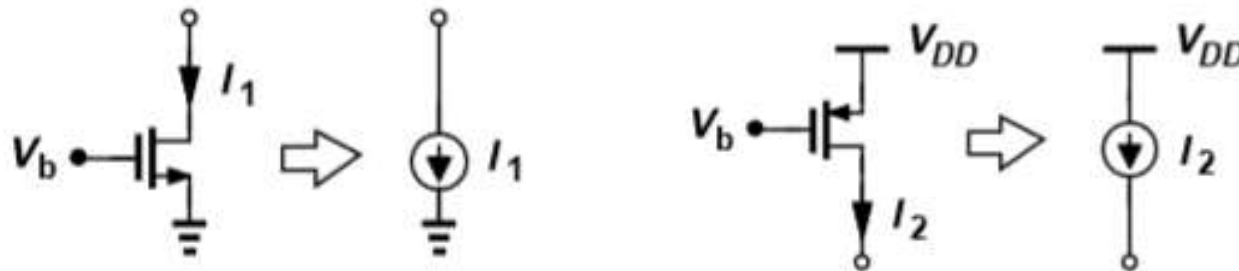
- **A saturated MOSFET can be used as a current source connected between the drain and the source.**

- **NMOS current sources inject current into ground while PMOS current sources draws current from V_{DD}.**

Derivation of I/V Characteristics
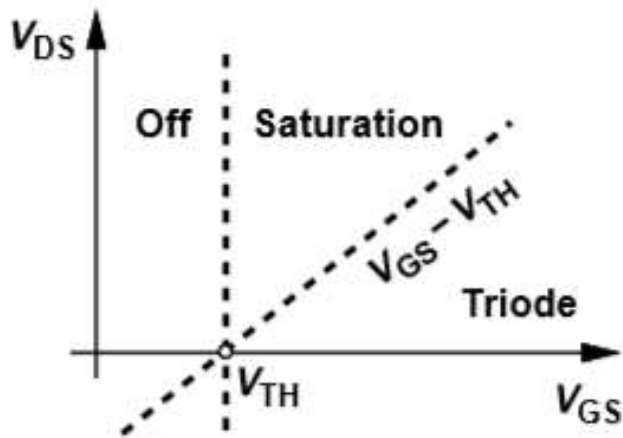
- $V_{DS} = V_{GS} - V_{TH} = V_{D,sat}$ **is the line between saturation and triode region.**
- **For a given V_{DS}, the device eventually leaves saturation as V_{GS} increases.**
- **The drain is defined as the terminal with a higher (lower) voltage than the source for an NMOS (PMOS).**
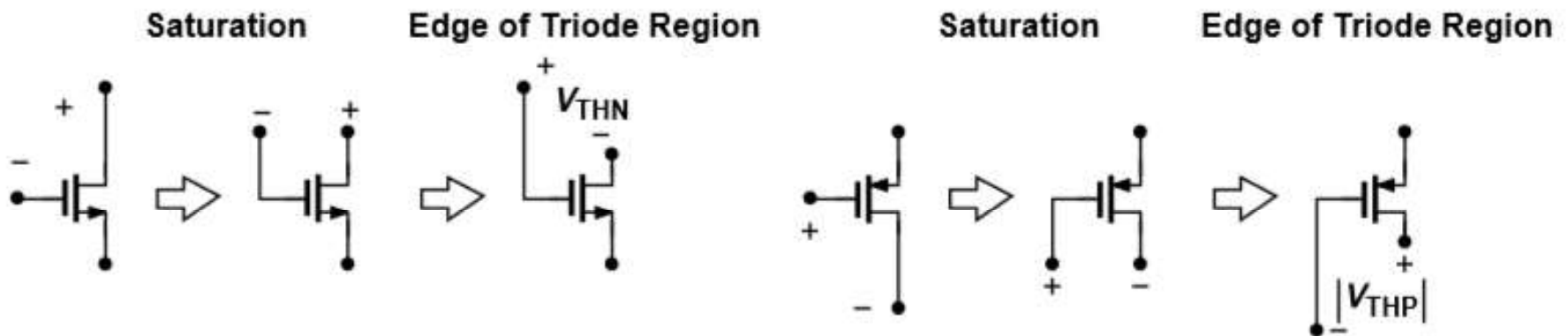
MOS Transconductance

$$g_m = \frac{\partial I_D}{\partial V_{GS}}\bigg|_{VDS \text{ const.}} = \mu_n C_{ox} \frac{W}{L}(V_{GS} - V_{TH})$$

$$= \sqrt{2\mu_n C_{ox} \frac{W}{L} I_D} = \frac{2I_D}{V_{GS} - V_{TH}}.$$

- **Transconductance (usually defined in the saturation region) is defined as the change in drain current divided by the change in the gate-source voltage.**

- **g_m represents the sensitivity of the device since a high value implies a small change in V_{GS} will result in a large change in I_D.**

- **Transconductance in saturation region is equal to the inverse of R_{on} in the deep triode region.**

MOS Transconductance

g_m vs $V_{GS} - V_{TH}$, W/L Constant | g_m vs I_D, W/L Constant | g_m vs $V_{GS} - V_{TH}$, I_D Constant
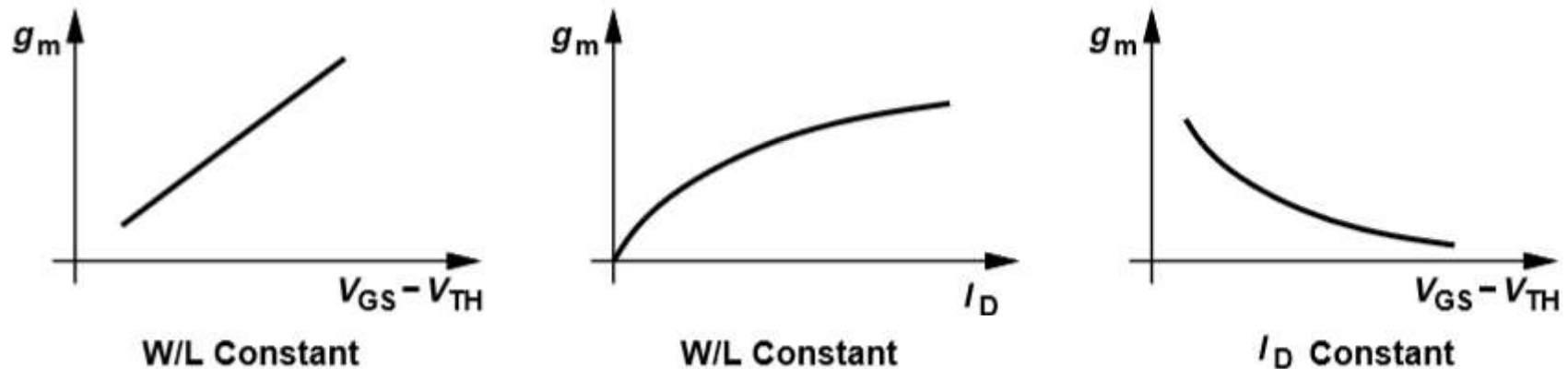
- **Each expression for transconductance is useful in studying its behavior.**

- **Drain current and overdrive voltage are *bias* values.**

- **If a small signal is applied to a device with defined bias values, we assume the signal amplitude is small enough that the variation in transconductance is negligible.**
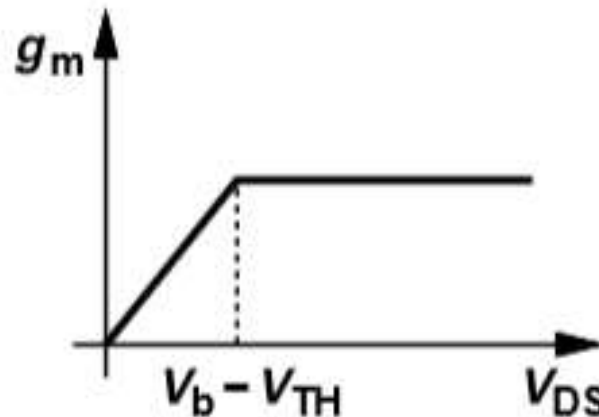
MOS Transconductance
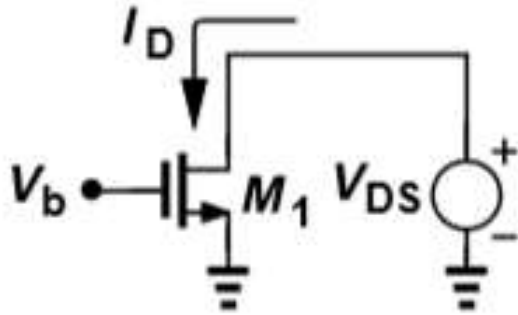
- **To find the transconductance for the topology on the left with respect to V_{DS},**

  - So long as $V_{DS} \geq V_b - V_{TH}$, M_1 is in saturation, so I_D is relatively constant, and therefore so is g_m.

  - When M_1 enters triode region $(V_{DS} < V_b - V_{TH})$,

$$g_m = \frac{\partial}{\partial V_{GS}} \left\{ \frac{1}{2} \mu_n C_{ox} \frac{W}{L} \left[2(V_{GS} - V_{TH})V_{DS} - V_{DS}^2 \right] \right\} = \mu_n C_{ox} \frac{W}{L} V_{DS}.$$

MOS Transconductance

- **For PMOS,**

$$g_m = -\mu_p C_{ox}(W/L)(V_{GS} - V_{TH})$$
$$= -2I_D/(V_{GS} - V_{TH})$$
$$= \sqrt{2\mu_p C_{ox}(W/L)I_D}.$$